# P(r)oučavanje geometrije u srednjoj školi

## 10. kongres nastavnika matematike Republike Hrvatske

## Zagreb - PMF, 1. i 2. srpnja 2024.

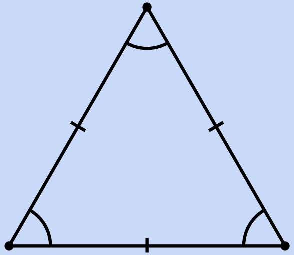
Vlatka Hižman-Tržić, prof. savjetnica, Tehnička škola Virovitica

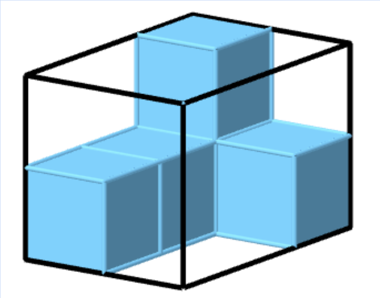
# Uvod

- geometrija većem dijelu učenika dio matematike koji najmanje vole i razumiju
- u srednju školu uglavnom dolaze sa slabom usvojenosti temeljnih koncepata geometrije, miskoncepcijama i „bore” se s terminologijom

1)

$P_{\Delta} = a \cdot b \cdot c$

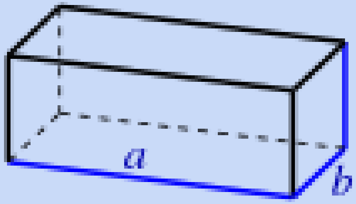


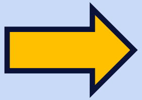


površina trokuta = obujam kvadra

2)

skiciraj kvadar



**Cilj moga izlaganja**: osvijestiti i analizirati kako pomoći učenicima da usvoje osnovne geometrijske pojmove i koncepte te „razbiti“ njihove miskoncepcije u geometriji

**Supružnici van Hiele** (50-tih godina prošloga stoljeća) pokušavali su otkriti razloge lošeg uspjeha u učenju geometrije i doći do konkretnih metoda kojima bi takvo stanje popravili.

Van Hiele-ova teorija:

**Učenje geometrije bit će djelotvorno ako su učenici aktivno uključeni u istraživanje geometrijskih objekata u ravnini i prostoru te iznose vlastita opažanja o obliku, svojstvima i vezama koje su uočili.**

# Kako poučavam geometriju u svojoj nastavnoj praksi ?

- u svome poučavanju geometrije polazim od predznanja učenika

- primjenjujem vizualizaciju (posebice GeoGebra aplete, ali i modele za stereometriju)

**površina trokuta** (GGB Š. Šuljić) samo pomoću površine pravokutnika (za učenike iz uvoda)

- inzistiram na preciznom definiranju i razlikovanju geometrijskih likova te prepoznavanju i primjeni njihovih svojstava u problemskim zadatcima

- ne propustim navesti učenike na provođenje jednostavnijih dokaza:

**zbroj unutrašnjih kutova trokuta** (GGB D. Belavić), Euklidovog poučka o katetama i visini u pravokutnom trokutu, sinusovog i kosinusovog poučka

# Primjeri :

1) **učinimo geometriju zornom** – modeli i primjena GeoGebre u radnim listićima za učenike

**Poliedri RM** **Stožac RM**

Rotacijska tijela - stožac RM

**Aktivnost 1:** Ulazna kartica

1. Ako je plašt valjka kvadrat, tada je duljina njegova promjera jednaka duljini njegove visine. T N
2. Koliki je obujam tijela koje nastaje rotacijom pravokutnika sa stranicama 4 cm i 7 cm oko njegove dulje stranice ?
3. Kocku obujma 65 $cm^3$ uronimo u vodu koja se nalazi u valjkastoj čaši promjera otvora 6 cm i visine 8 cm. Za koliko se centimetara podigla razina vode u čaši ?

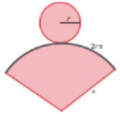
**Aktivnost 2:** Upoznajmo stožac !

Pogledajte zašto je stožac rotacijsko tijelo te kako nastaje rotacijom na poveznici https://www.geogebra.org/m/yGhwuYYF .
Upoznajte stožac i njegove elemente u na poveznici https://www.geogebra.org/m/ptwywhcd , te popunite praznine i odgovorite na pitanje.

Stožac je ____________ __________ omeđeno jednom ____________ i ______________ ________ .
**Uspravni stožac** nastaje rotacijom ______________ __________ oko jedne njegove ___________ .
Visina stošca je ________________________________________________________________ .
Izvodnica stošca je ______________________________________________________________ .
Os stošca je ___________________________________________________________________ .
Karakteristični trokut stošca čine __________ _________ , ___________ i ___________ .
Osni presjek stošca je _________ stošca ravninom ____________ na _______ , a sadrži ____ ________ .
Plašt stošca je __________ ______________ polumjera ____ .
Os **kosog stošca** nije __________ na ravninu osnovke.
Je li kosi stožac rotacijsko tijelo ?

**Aktivnost 3:** Izračunajmo oplošje i obujam stošca !

Skicirajte u bilježnicu mrežu stošca i prisjetite se kako smo računali površinu kružnoga isječka na 2 načina.



$P =$
$P =$

Izjednačite ta dva izraza za površinu isječka i dobit ćete izraz za kut u mreži stošca. $\alpha =$

Zapišite izraz za računanje oplošja stošca.

Skicirajte uspravni stožac i njegov osni presjek te uočite kut pri vrhu osnog presjeka uspravnog stošca.
Pomoću koje trigonometrijske funkcije bismo izračunali taj kut ? Zapišite izraz.

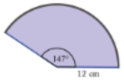
Pogledajte kratki video o odnosu volumena valjka i stošca i zapišite kako računamo obujam stošca.
https://drive.google.com/file/d/17R8B1Cngj4zEBiHzWW56UKrwzIFfemgS/view?usp=share_link

**Aktivnost 4:** Riješimo zadatke sa stošcem !

Zadatak 1: Deset hrpi pijeska u obliku stošca visine 2m i dijametra baze 2m želimo presuti u vreće. Koliko nam vreća za to treba ako u jednu vreću stane 120 L pijeska ?

Zadatak 2: Kornet za sladoled ima oblik stošca s izvodnicom 15 cm i promjerom osnovke 9 cm. Koliko se najviše $cm^3$ sladoleda može staviti u kornet ? Koliki je kut pri vrhu osnog presjeka korneta?

Zadatak 3: Koliki je obujam stošca čiji je plašt prikazan na slici ?




**Aktivnost 5:** Vrednovanje za učenje
https://quizizz.com/admin/quiz/6469e041a44914001ecd3a39/sto%C5%BEac?searchLocale=

2) **učinimo geometriju razumljivijom** – podrška učeniku u rješavanju problema u planimetriji i stereometriji – nastavnik „glumi” potporanj, skelu kako bi pomogao učenicima postići višu razinu razumijevanja i stjecanje vještina

Primjer 1: propitivanje učenika – nastavnik postavlja pitanja s naglaskom na kritične točke u rješavanju problema
Ž. Dijanić, 2017. Matematika i škola

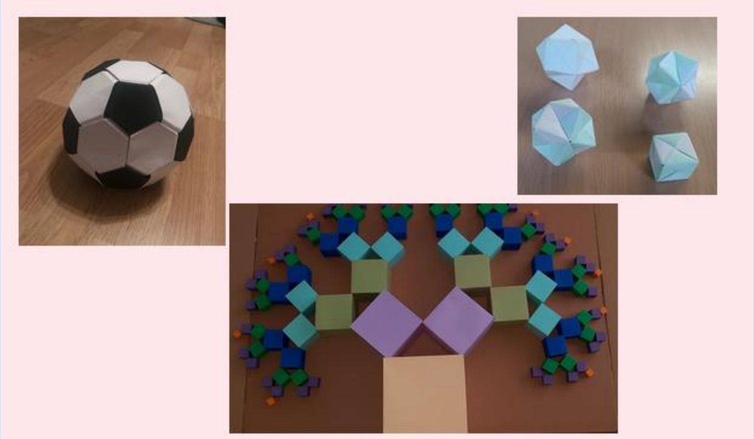
Primjer 2: vođena potpitanja u prilično kompleksnom zadatku presjeka kocke ravninom (uz pomoć GeoGebre)

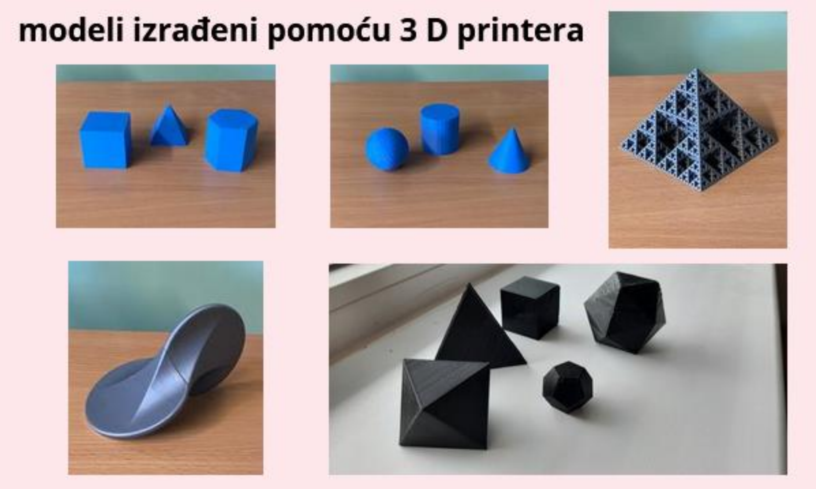
***scaffolding***

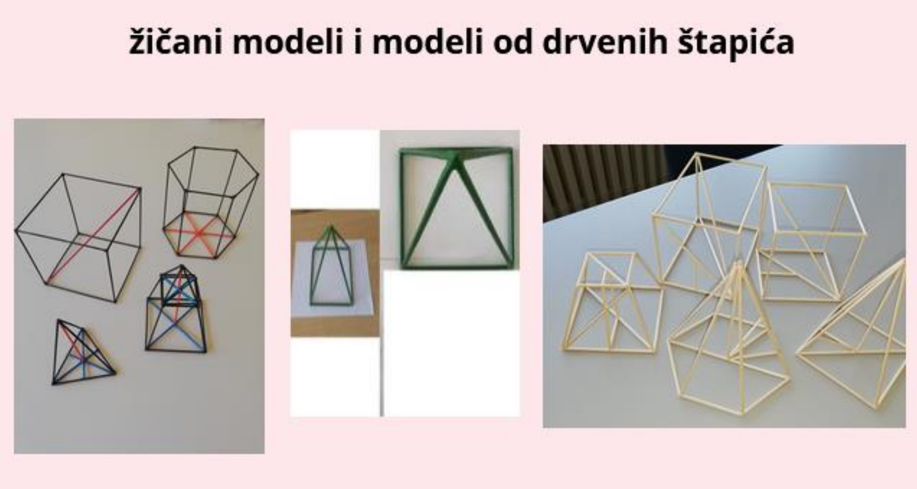
3) **učinimo geometriju zanimljivom i kreativnom** – sudjelovanje u eTwinning projektima

**Vizualna matematika,** 2021.

- poticanje učenika na izradu geometrijskih tijela i krivulja na više različitih načina
- poticanje učenika na osmišljavanje zadataka koji se temelje na skicama, slikama i modelima

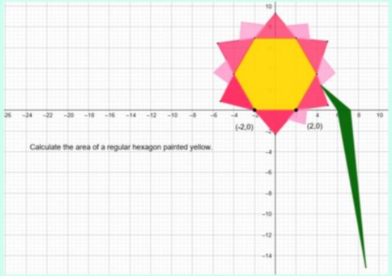


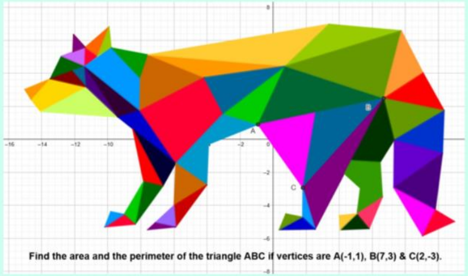


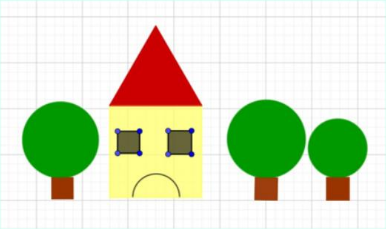



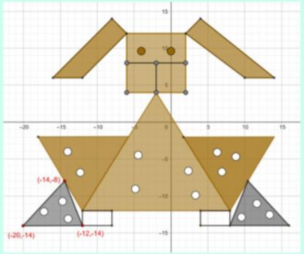
**Math Travellers, šk.god.** 2021./2022.

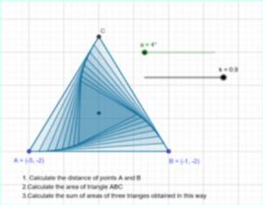
- kreativni radovi učenika primjenom matematičkih zakonitosti, posebno geometrije uz GeoGebru

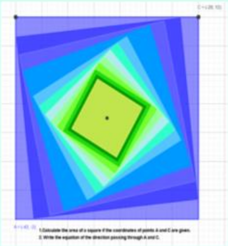



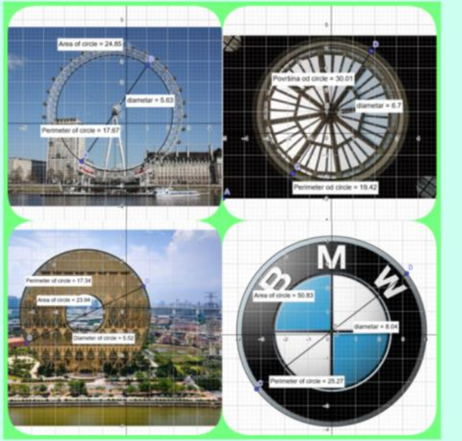



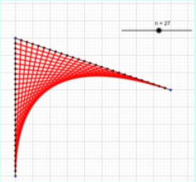


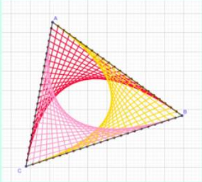



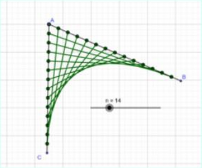



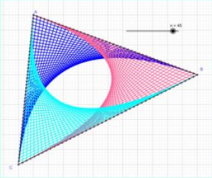


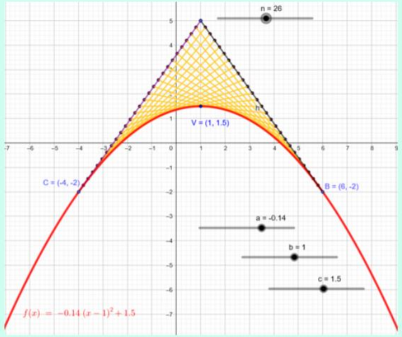

## String art & Parabola







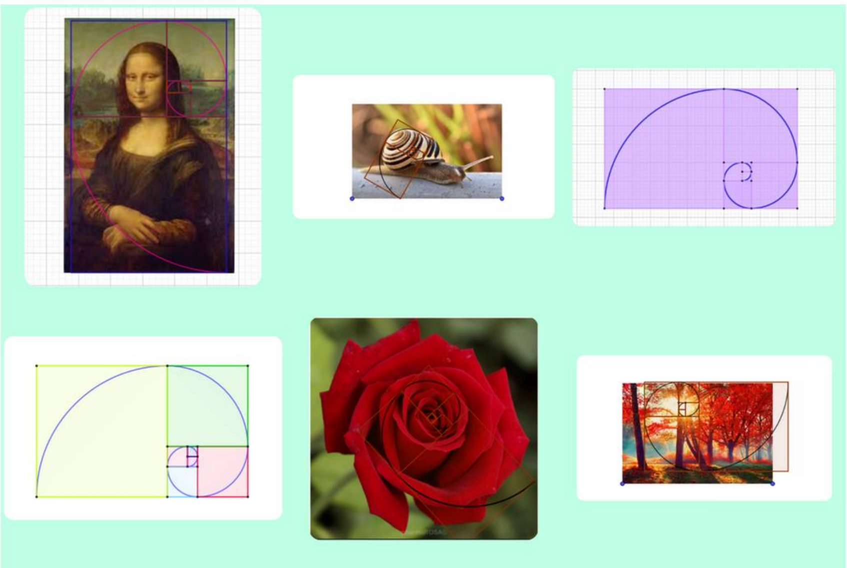


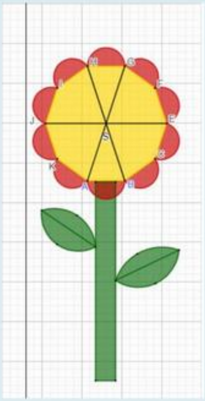



String Art - radni list

# G.A.M.E, šk.god. 2022./2023.

- učenici su proučavali zlatni rez, simetriju i primjenu geometrije u svakodnevnom životu





1. Calculate the area of the golden triangle ABS if the length of AB is 1 cm.
2. Calculate the radius of the circumscribed circle of a regular decagon.

*Manuel Bošnjak*
*Technical School Virovitica, Croatia*

1. $A_{ABS} = \frac{AB^2 \cdot sinA \cdot sinB}{2sinC}$

$$\measuredangle A = 72^0 = \measuredangle B; \measuredangle ASB = 36^0$$

$$A_{ABS} = \frac{1 \cdot sin72^0 \cdot sin72^0}{2sin36^0}$$

$$sin72^0 = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$sin36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$A_{ABS} = \frac{\frac{10+2\sqrt{5}}{16}}{2 \cdot \frac{\sqrt{10-2\sqrt{5}}}{4}} = \frac{10+2\sqrt{5}}{16} \cdot \frac{2}{\sqrt{10-2\sqrt{5}}}$$

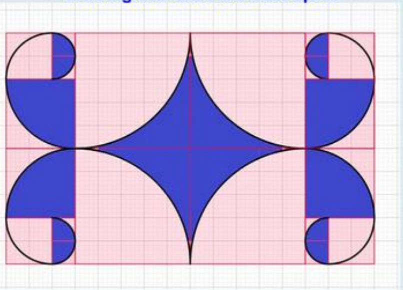
$$A_{ABS} = \frac{10+2\sqrt{5}}{8\sqrt{10-2\sqrt{5}}}$$

2. The radius is AS; SAB is a golden triangle, so

$$AS = AB \cdot \varphi \Rightarrow AS = \varphi$$

*Maria*
*Jean Monnet High School*

**Drawing with the Fibonacci spiral**



In the figure above, there are four Fibonacci spirals in which the largest square has side 5.

a) Find the area of the largest Golden rectangle.
b) Find the blue area

*Andrei T*
*Jean Monnet High School*

a) The largest golden rectangle is a whole large rectangle because its sides are 16 units and 10 units, and their ratio is 1.6 the golden number. The area of that rectangle is

P=16x10 =160 square units

b) The blue area is equal to the sum of the area of a circle with a radius of 3 units and the area of two circles with a radius of 1 unit, and the area of a circle with a radius of 5 units subtracted from the area of a square with a side of 10 units

$$P = 3^2\pi + 2x1^2\pi + 10^2 - 5^2\pi$$

$= 100 - 14\pi$

=56.02 square units.

*Luka Kaselj*
*Technical School Virovitica, Croatia*

## Symmetry in real life

Technical School Virovitica, Croatia

Student: Patrik Živković

Pejačević Castle is located in the center of Virovitica. It is one of a series of Slavonian castles of the famous Pejačević noble family, who received the noble title "Pejačevići Virovitički" precisely because of this estate. The castle was built between 1800 and 1804 according to the plans of the Viennese architect N. Roth. The castle is symmetrical with respect to the axis passing through its center.

The proposed problem

What is the volume of the two wings of a cube-shaped castle with a length of 7.5 m, a width of 3 m and a height of 4.5 m without a roof ?

Solution

The volume of two parts of the castle in the form of a cube is:

$P = 7.5m \times 3m \times 4.5m \cdot 2$

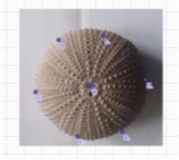
$P = 202.5\ m^3$

Technical School Virovitica, Croatia

Student: Hrvoje Mijok

This is a natural sea urchin shell from the Croatian Adriatic Sea.

It is centrally symmetric with respect to its center.



The proposed problem

Calculate an area of an empty shell with a diameter of 8 cm when viewed from above ?

Solution

We can roughly assume that the shell is a circle when viewed from above and its area is equal to:

$P = 4^2 \cdot \pi cm^2$

$P = 50.27 cm^2$

Growing with Applied Mathematics Experiences (G.A.M.E.)

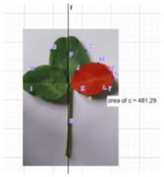
Symmetry in real life

Technical School Virovitica, Croatia

Student: Gabrijel Bakić

This is a picture of a three-leaf clover growing in meadows and lawns.

This plant is axisymmetric with respect to the direction that passes through its center.




The proposed problem

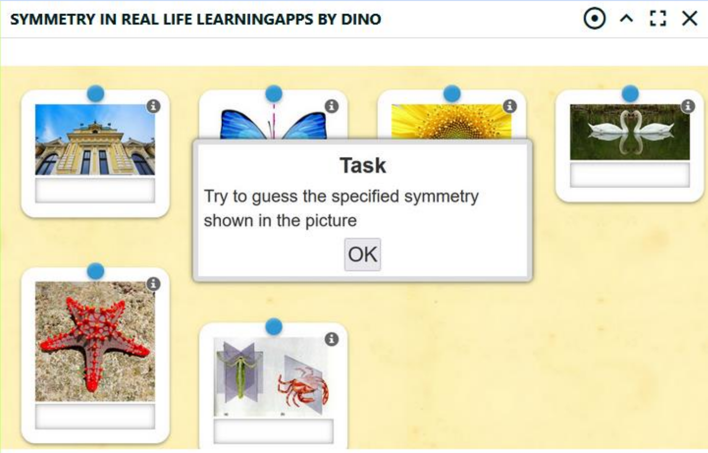
What is the area of one clover leaf if the length in the middle is 3.2 cm and the width is 1.6 cm ?

Solution

One clover leaf has the shape of an ellipse whose main semi-axis is equal to 1.6 cm, and the other semi-axis is 0.8 cm, so the area is equal to:

$P = 1.6cm \cdot 0.8cm \cdot \pi$

$P = 4.02cm^2$

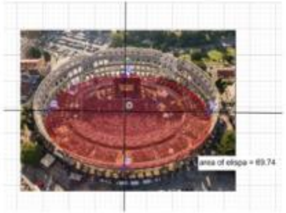



## Symmetry in real life

Technical School Virovitica, Croatia

Student: Dino Špondreht

The amphitheater in Pula or Pula Arena (popularly known as Divić-grad) is the largest and best-preserved monument of ancient architecture in Croatia. It ranks 6th in size among Roman amphitheaters in the world, and is the only one in the world that has all three Roman architectural orders completely preserved. It is surprising that the Arena, as an indisputable cultural treasure of Croatia, but also of the world, is not yet on the UNESCO World Heritage List. The Pula arena is symmetrical with respect to the axis of the ellipse.




The proposed problem

What is the area of the base of the amphitheater if the longer axis is 132.45 m and the shorter axis is 105.10 m ?

Solution

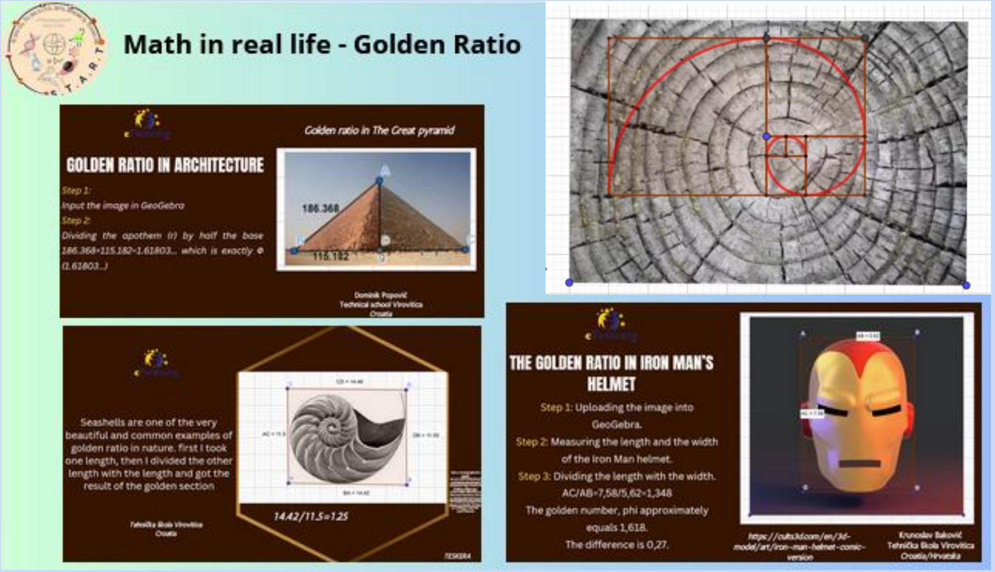
Pula Arena is an exceptionally geometrically regular building, it has an elliptical appearance so the area P is:

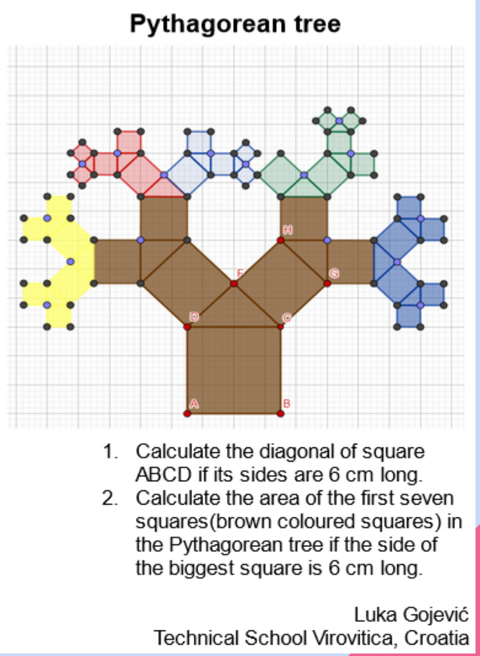
$P = a \times b \times \pi = \frac{132.45}{2} m \cdot \frac{105.10}{2} m \cdot \pi$

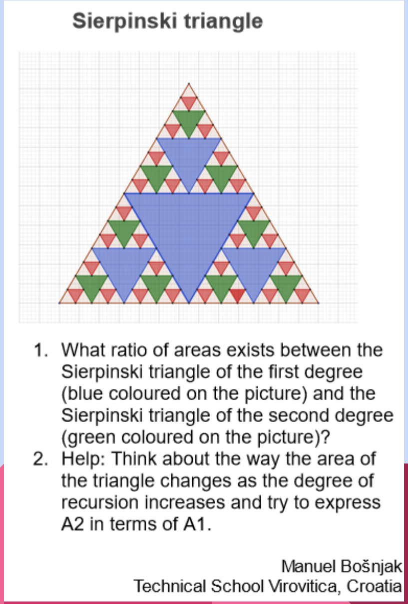
$P = 10933.13\ m^2$

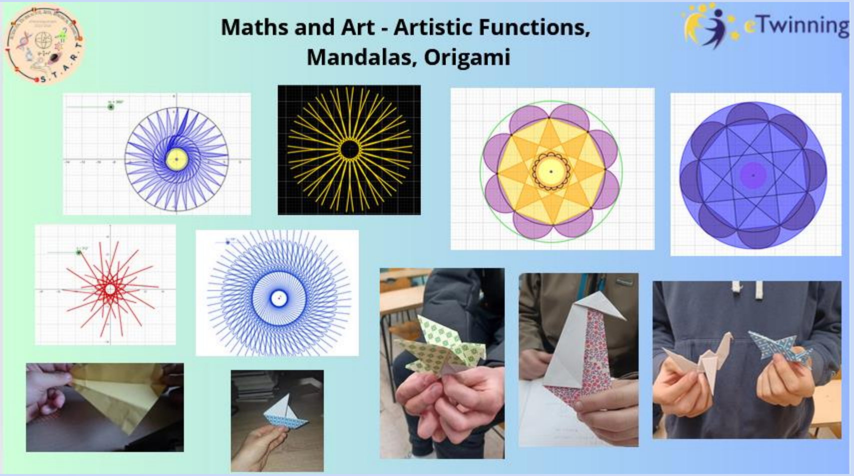
**S.T.A.R.T, šk.god.** 2023./2024.

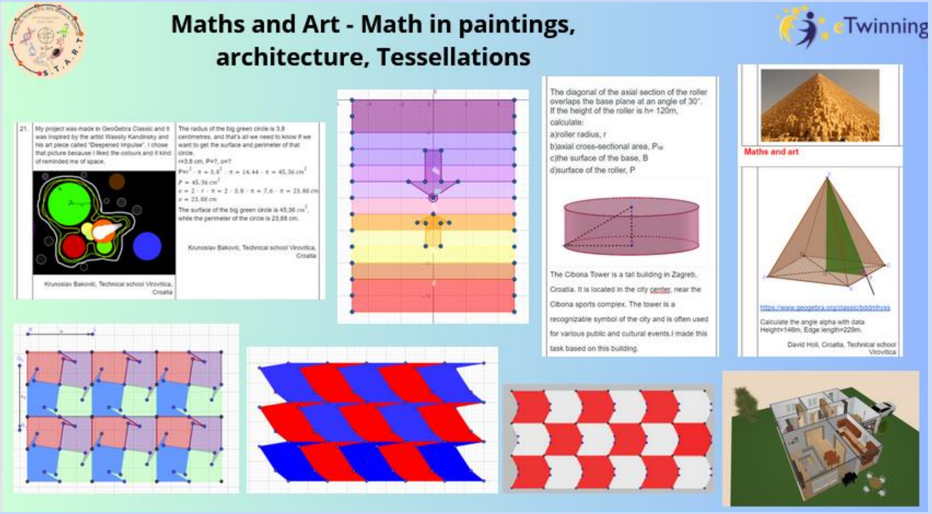
- kroz proučavanje znanosti, tehnologije i umjetnosti uz matematiku (posebno geometriju) kao temeljni alat i poveznicu s drugim znanostima, učenici su proučavali pojave iz stvarnog života

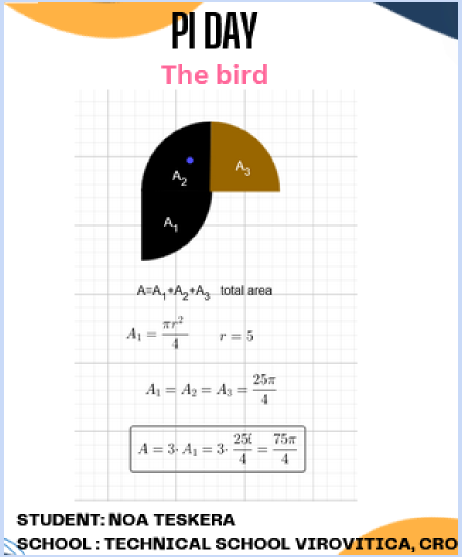


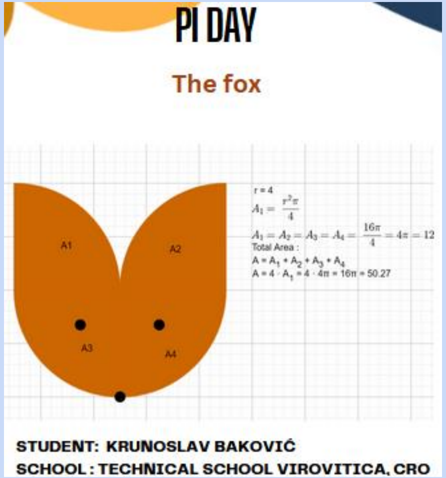



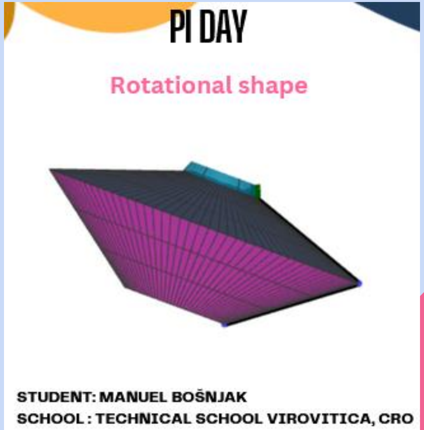



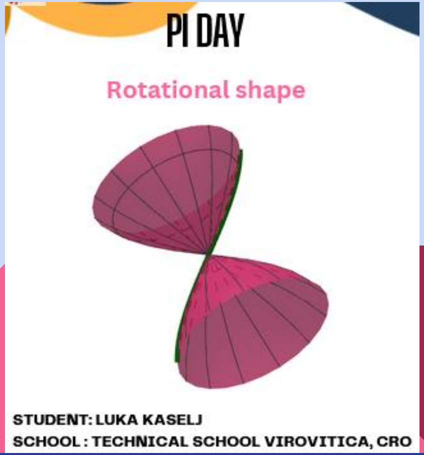

Maths and Art - Artistic Functions, Mandalas, Origami
eTwinning
S.T.A.R.T


Maths and Art - Math in paintings, architecture, Tessellations
eTwinning
S.T.A.R.T
Maths and art

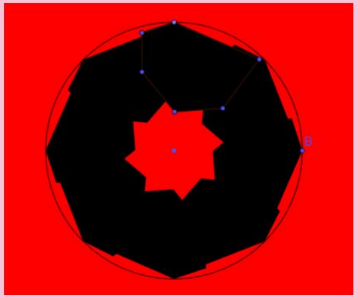

PI DAY
The bird
A1
A2
A3
A=A1+A2+A3 total area
$A_1 = \frac{\pi r^2}{4}$ $r = 5$
$A_1 = A_2 = A_3 = \frac{25\pi}{4}$
$A = 3 \cdot A_1 = 3 \cdot \frac{25(}{4} = \frac{75\pi}{4}$
STUDENT: NOA TESKERA
SCHOOL : TECHNICAL SCHOOL VIROVITICA, CRO


PI DAY
The fox
A1
A2
A3
A4
r = 4
$A_1 = \frac{r^2\pi}{4}$
$A_1 = A_2 = A_3 = A_4 = \frac{16\pi}{4} = 4\pi = 12$
Total Area :
A = A1 + A2 + A3 + A4
A = 4 · A1 = 4 · 4π = 16π = 50.27
STUDENT: KRUNOSLAV BAKOVIĆ
SCHOOL : TECHNICAL SCHOOL VIROVITICA, CRO

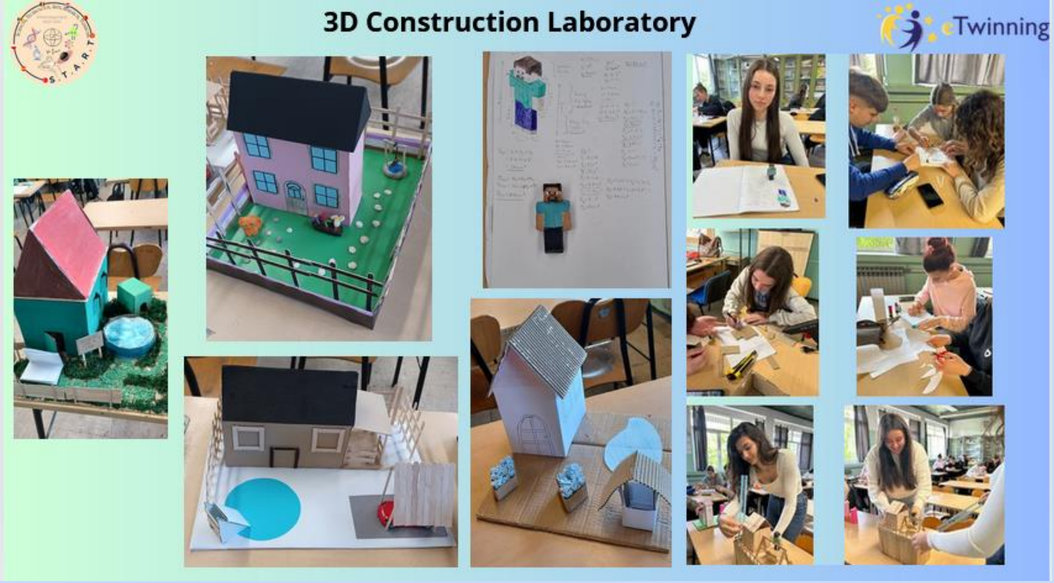

PI DAY
Rotational shape
STUDENT: MANUEL BOŠNJAK
SCHOOL : TECHNICAL SCHOOL VIROVITICA, CRO


PI DAY
Rotational shape
STUDENT: LUKA KASELJ
SCHOOL : TECHNICAL SCHOOL VIROVITICA, CRO

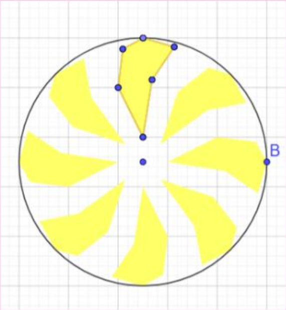
Have a great summer break and keep yourselves safe!



From Krunoslav Baković
Technical school Virovitica

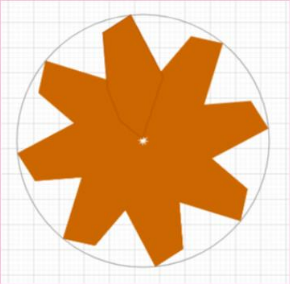



All the best and happy holidays



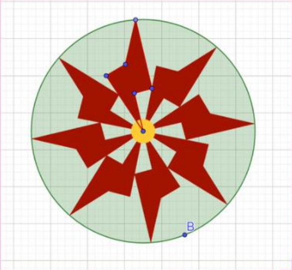

Teskera, Tehnička škola Virovitica

Thank you all for this beautifull experience!



*From Dominik Popović*
*Technical School Virovitica*

Enjoy your summer vacation and rest!




From Manuel Bošnjak
Technical school Virovitica

## 4) učinimo geometriju primjenjivom

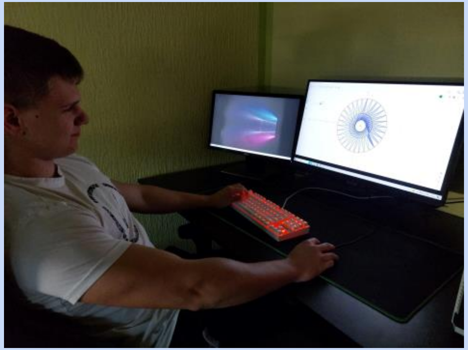
– različite aktivnosti u kojima učenici uočavaju svrhu i primjenu geometrije

**eBook Stereometrija** → 20 matematičkih problema vezanih za računanje obujma ili oplošja geometrijskih tijela koje su prikupili učenici, a zadani su pomoću skica ili crteža

**Eratostenov eksperiment** → knjiga u alatu Storyjumper o eksperimentu mjerenja opsega Zemlje pomoću sjene štapa na dan ekvinocija

# Zaključak

U poučavanju geometrije treba slijediti promišljanja van Hiele-ovih i učenike dobro osmišljenim aktivnostima poticati da izrađuju, skiciraju, predviđaju, interpretiraju, uspostavljaju veze među različitim zapisima što ih, uz nužnu sistematičnost zapisa, dovodi do uspjeha u rješavanju geometrijskih problema i usvajanju planiranih ishoda.



# Evaluacija

https://bit.ly/3xjx8wt


vlatka.hizman-trzic@skole.hr