

O nastavi računarstva na PMF–MO (i malo na FER–u)

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PMF–Matematički odsjek
Sveučilište u Zagrebu

31. ožujak 2011.

O čemu zapravo imam namjeru pričati?

"Kriv" je prof. Andrej Bauer iz Ljubljane,

i naravno Vesna Ž.: "Što, zapravo, predaješ studentima FER-a?"

Je li došao trenutak da počnemo razgovarati o zajedničkom studiju računarstva (eng. computer science) PMF-a, FER-a, FSB-a, IRB-a, ...?

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Moja nastava na FER-u

Na FER-u dvije posljednje godine držim nastavu iz kolegija
Matematička logika i izračunljivost.

To je izborni kolegij na diplomskom studiju **Računarstvo; profil:
Računarska znanost**

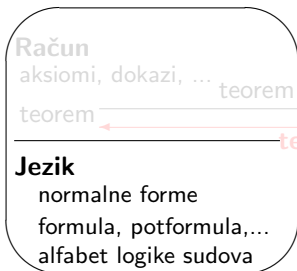
Sadržaj kolegija Mat. logika i izračunljivost (FER) (1)

1. ciklus (5 tjedana): **Logika sudova**

- 1 skupovi; sintaksa i semantika
- 2 normalne forme; glavni test
- 3 račun sudova RS ("dokaz" teorema potpunosti)
- 4 **modalna propozicionalana logika**

Sadržaj kolegija Mat. logika i izračunljivost (FER) (2)

SINTAKSA



teorem adekvatnosti

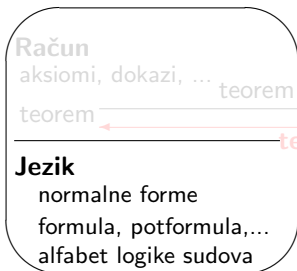
teorem potpunosti

SEMANTIKA

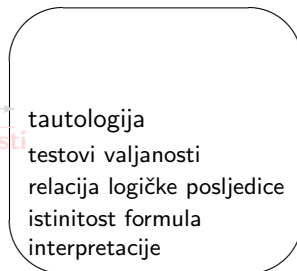


Sadržaj kolegija Mat. logika i izračunljivost (FER) (2)

SINTAKSA



SEMANTIKA



teorem adekvatnosti

teorem potpunosti

Sadržaj kolegija Mat. logika i izračunljivost (FER) (2)

SINTAKSA

Račun

aksiomi, dokazi, ...

teorem

Jezik

normalne forme

formula, potformula, ...

alfabet logike sudova

teorem adekvatnosti

teorem potpunosti

SEMANTIKA

tautologija

testovi valjanosti

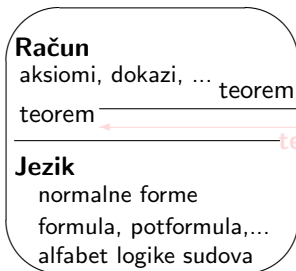
relacija logičke posljedice

istinitost formula

interpretacije

Sadržaj kolegija Mat. logika i izračunljivost (FER) (2)

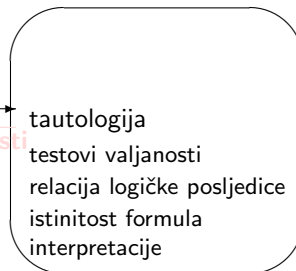
SINTAKSA



teorem adekvatnosti

teorem potpunosti

SEMANTIKA



Sadržaj kolegija Mat. logika i izračunljivost (FER) (2)

SINTAKSA

Račun

aksiomi, dokazi, ... teorem adekvatnosti

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teorem potpunosti

Sadržaj kolegija Mat. logika i izračunljivost (FER) (3)

2. ciklus (4 tjedna): **Logika prvog reda**

- 1 sintaksa i semantika
- 2 preneksna normalna forma
- 3 glavni test
- 4 formalni sistem

Sadržaj kolegija Mat. logika i izračunljivost (FER) (4) – Primjer

Primjer 1

Ispitajmo valjanost formule $(\forall xA(x) \rightarrow \exists xB(x)) \rightarrow \exists x(A(x) \rightarrow B(x))$

Na sljedećoj slici dano je jedno stablo glavnog testa za zadanu formulu.

$$(\forall x A(x) \rightarrow \exists x B(x)) \rightarrow \exists x (A(x) \rightarrow B(x)) \quad \perp$$

$$\forall x A(x) \rightarrow \exists x B(x) \quad \top$$

$$(*) \quad \exists x (A(x) \rightarrow B(x)) \quad \perp$$

$$\forall x A(x) \quad \perp \quad (\text{..a..})$$

$$\exists x B(x) \quad \top \quad (\text{..b..})$$

$$A(a) \quad \perp$$

$$B(b) \quad \top$$

$$(*) \quad \exists x (A(x) \rightarrow B(x)) \quad \perp \quad \textcircled{a}$$

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$$A(b) \rightarrow B(b) \quad \perp$$

$$A(a) \quad \top$$

$$A(b) \quad \top$$

$$B(a) \quad \perp$$

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$$(\forall x A(x) \rightarrow \exists x B(x)) \rightarrow \exists x (A(x) \rightarrow B(x)) \quad \textcircled{\perp}$$

$$\forall x A(x) \rightarrow \exists x B(x) \quad \textcircled{\top}$$

$$(*) \quad \exists x (A(x) \rightarrow B(x)) \quad \perp$$

$$\forall x A(x) \quad \textcircled{\perp} \quad (\text{..a..})$$

$$\exists x B(x) \quad \textcircled{\top} \quad (\text{..b..})$$

$$A(a) \quad \textcircled{\perp}$$

$$B(b) \quad \textcircled{\top}$$

$$(*) \quad \exists x (A(x) \rightarrow B(x)) \quad \perp \quad \textcircled{a}$$

$$(*) \quad \exists x (A(x) \rightarrow B(x)) \quad \perp \quad \textcircled{b}$$

$$A(a) \rightarrow B(a) \quad \textcircled{\perp}$$

$$A(b) \rightarrow B(b) \quad \textcircled{\perp}$$

$$A(a) \quad \textcircled{\top}$$

$$A(b) \quad \textcircled{\top}$$

$$B(a) \quad \textcircled{\perp}$$

$$B(b) \quad \textcircled{\perp}$$

X

X

Sadržaj kolegija Mat. logika i izračunljivost (FER) (5)

3. ciklus (4 tjedna): **Izračunljivost**

- 1 RAM–stroj
- 2 parcijalno rekurzivne funkcije
- 3 kodiranje; indeksi; Kleenijev teorem
- 4 Churchova teza; Churchov teorem

Što nažalost ne uspijem napraviti na FER-u?

- 1 rezolucija (logičko programiranje i automatski dokazivači)
- 2 Herbrandov teorem
- 3 λ -račun (funkcijsko programiranje)
- 4 temporalna logika; logike znanja i vjerovanja
- 5 složenost algoritama; problem SAT

Naslovi nekih studentskih seminara na FER-u

- 1 Halting problem
- 2 ABAK-računalo
- 3 Jednosmjerne funkcije
- 4 λ -račun
- 5 Temporalna logika
(možda zajednički stručni članak jedne studentice FER-a i jednog studenta PMF-MO)

Završni i diplomski radovi studenata FER-a

- 1 H. Bandov, Problem PRIMES (završni rad, 2010.)
- 2 H. Bandov, Algoritmi sa slučajnim elementima (diplomski)
- 3 M. Mikša, Deskriptivna teorija složenosti (diplomski)
- 4 T. Novak, Gödelovi teoremi nepotpunosti (diplomski)

Moja nastava na PMF–MO

- **Teorija skupova**
- Matematička logika
- Izračunljivost
- Složenost algoritama
- **Logika i računarstvo** (poslijediplomski studij matematike).

Moja nastava na PMF–MO

- **Teorija skupova**
- **Matematička logika**
- Izračunljivost
- Složenost algoritama
- **Logika i računarstvo** (poslijediplomski studij matematike).

Moja nastava na PMF–MO

- **Teorija skupova**
- **Matematička logika**
- **Izračunljivost**
- Složenost algoritama
- **Logika i računarstvo** (poslijediplomski studij matematike).

Moja nastava na PMF–MO

- **Teorija skupova**
- **Matematička logika**
- **Izračunljivost**
- **Složenost algoritama**
- **Logika i računarstvo** (poslijediplomski studij matematike).

Moja nastava na PMF–MO

- **Teorija skupova**
- **Matematička logika**
- **Izračunljivost**
- **Složenost algoritama**
- **Logika i računarstvo** (poslijediplomski studij matematike).

"Malo povijesti" ili kako je sve počelo

Povjerenstvo za izradu (sadašnjeg) plana na PMF–MO (2004.):
B. Dalbelo–Bašić, R. Manger, G. Nogo, D. Rosenzweig, M. Vuković

Stručni članak za ITI 2006: [A New MSc Curriculum in Computer Science and Mathematics at the University of Zagreb](#)

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Povjerenstvo za izradu (sadašnjeg) plana na PMF–MO (2004.):
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Stručni članak za ITI 2006: **A New MSc Curriculum in Computer Science and Mathematics at the University of Zagreb**

Sadašnji program diplomskog studija Matematika i računarstvo na PMF–MO

1. godina

Nastavnik	Kolegij	zimski		ljetni	
		P	V	P	V
Ribarić, S.	Građa računala	2	1		
Singer, S.	Oblikovanje i analiza algoritama	2	1		
Nogo, G.	Umjetna inteligencija	2	1		
Vuković, M.	Matematička logika	2	2		
Rogina, M.	Računalna grafika	2	1		
	Izborni predmet 1	2	1		
Jelenković, L.	Operacijski sustavi			2	1
Vuković, M.	Izračunljivost			2	1
Manger, R.	Baze podataka			2	1
Starčević, M.	Objektno programiranje (C++)			2	2
Grubišić, L.	Računarski praktikum 2			1	2
	Izborni predmet 2			2	1

2. godina

Nastavnik	Kolegij	zimski		ljetni	
		P	V	P	V
Manger, R., Puljić, K.	Interpretacija programa	2	1		
Manger, R.	Softversko inženjerstvo	2	1		
Franušić, Z.	Kriptografija i sigurnost mreža	2	1		
Grubišić, L.	Distribuirani procesi	2	1		
	Izborni predmet 3	3			
	Izborni predmet 4	3			
Vuković, M.	Složenost algoritama			2	1
Starčević, M.	Računarski praktikum 3			1	3
	Izborni predmet 5			2	1
	Izborni predmet 6			2	1
	Diplomski rad				

Vanjski suradnici na PMF–MO na studiju računarstva

- Leonardo Jelenković (FER), Operacijski sustavi
- Mladen Mauher, Društveni aspekti inform.–kom. tehnologije;
Upravljanje softverskim projektima
- Slobodan Ribarić (FER), Građa računala
- Sanja Singer (FSB), Uvod u paralelno računanje; Primjena paralelnih računala
Vedran Novaković (FSB) – vježbe iz oba kolegija
- Neva Slani (FSB), Formalne metode u računarstvu; Matematička logika u računarstvu
- Tomislav Šmuc (IRB), Strojno učenje
Matko Bošnjak (IRB) – vježbe

Nastavnici PMF–MO kao vanjski suradnici na FER–u na studiju računarstva

- 1 A. Dujella, Z. Franušić (PMF–MO), Diskretna matematika
- 2 R. Manger (PMF–MO), Programiranje
- 3 M.V. (PMF–MO), Matematička logika i izračunljivost

Kolegij na PMF–MO za studenta FER-a

M.V., Složenost algoritama

- ① 4. predavanje 07.03.2011 <https://acp.carnet.hr/p55297351/>
- ② 5. predavanje 14.03.2011 <https://acp.carnet.hr/p34965318/>
- ③ 6. predavanje 21.03.2011 <https://acp.carnet.hr/p32825104/>

Cambridge, Matko Botinčan (1)

M. Botinčan, Deskriptivna teorija složenosti: Verifikacija modela, magistarski rad, PMF–MO, 2005.

"U grubo: na **prve dvije studija se sadržajno obradi ono sto se na PMF–MO napravi na svih 3+2 godina iz računarstva**, plus još neke dodatne teme:

- 1 više arhitekture
- 2 više programiranja i algoritama
- 3 security
- 4 networking

Cambridge, Matko Botinčan (1)

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- 2 više programiranja i algoritama
- 3 security
- 4 networking

Cambridge, Matko Botinčan (2)

"Treća godina sadrži naprednije praktične i teorijske teme (dijelom one koje se kod nas rade na doktorskom studiju), te vlastiti projekt (ekvivalent našeg diplomskog rada; uz razliku što se ovdje traži da rad ima neki (barem malen) originalni doprinos, što u pravilu na kraju rezultira nekom publikacijom)."

Cambridge, Matko Botinčan (3)

"Mislim da je jedan od glavnih razloga zašto je computer science degree od Cambridgea visoko cijenjen na tržištu rada (barem u UK) upravo to što ovdje studenti po završetku trogodišnjeg studija **imaju ne samo direktno tržišno iskoristiva praktična znanja nego i vrlo čvrstu teorijsku podlogu.**

U odnosu na naš studij na PMF–MO tu je dodatno i:

- 1 teorija tipova
- 2 polimorfni lambda calculus
- 3 denotacijska semantika
- 4 logike i alati za specifikaciju i verifikaciju
- 5 csp/ccs/pi-racun
- 6 security prokoli.

Cambridge, Matko Botinčan (4)

"Ono što je posljedica ovakve strukture studija jest to da studenti u pravilu **NE upisuju master stupanj**.

Part III (ili MPhil) upisuje možda svega desetak posto studenata koji su završili Cambridge BA.

Ne stoga što se ne bi mogli kvalificirati za daljnji studij, nego zato što im za većinu poslova u IT&related struci to nije potrebno, te je BA titula dovoljno cijenjena da još jedna godina mastera ne daje nikakvu razliku.

Štoviše, upisivanje mastera se smatra pointless ukoliko se ne planira odlazak na PhD, tako da master/PhD upisuju u principu samo oni koji bi htjeli ostati u akademiji/researchu ili se baviti visoko specijaliziranim poslom u industriji (npr. verifikacija)."

Cambridge, Matko Botinčan (4)

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Cambridge, Matko Botinčan (5)

"Svakako podupirem inicijativu za prijedlogom organizacije jedinstvenog studija računarstva u sklopu Sveučilišta.

Tako studij računarstva funkcionira praktički na svim boljim sveučilištima u svijetu, pa zašto ne bi i u Zagrebu."

"Osobno sam uvjeren da uspješni computer science studij mora biti jak u oba segmenta - teorijskom i praktičnom, i to već nakon tri godine studija.

Iz tog razloga vidim da organizacija novog zajedničkog studija računarstva od strane PMF-MO i FER-a može biti veliki dobitak u odnosu na oblik studija koji trenutno postoji."

Cambridge, Matko Botinčan (6)

Ovdje mozete vidjeti kratke opise svih studijskih programa:

<http://www.cl.cam.ac.uk/intro/overview/node5.html>

Ovdje se nalaze sadrzaji pojedinih godina po predmetima:

Part IA: <http://www.cl.cam.ac.uk/teaching/1011/part1a-cst.html>

Part IB: <http://www.cl.cam.ac.uk/teaching/1011/part1b.html>

Part II: <http://www.cl.cam.ac.uk/teaching/1011/part2.html>

ACS: <http://www.cl.cam.ac.uk/teaching/1011/acs.html>

Edinburgh, Domagoj Vrgoč

Godina	Sadržaji		Istaknuto
1.	Informatika	Mat. Izborno	Computation; logic; data; programming; calculus; algebra
2.	Informatika	Mat.	Computer systems; algorithms and data structures; problem solving, learning and planning; geometry; probability
3.	Računarstvo		A wide choice of modules
4.	Računarstvo		A wide choice of modules

<http://www.ed.ac.uk/schools-departments/informatics/undergraduate/ourdegrees>

ETH–Zürich, Grgur Petric Maretić

G. Petric Maretić, Fuzzy logika, Diplomski rad, PMF–MO, 2010.

Preddiplomski studij na ETH:

Uvod u programiranje, Diskretna matematika, **Linearna algebra**,
Matematička analiza 1

Strukture podataka i algoritmi, Digitalni sklopovi, Paralelno
računarstvo, **Matematička analiza 2**, Fizika

Teorijsko računarstvo, Građa računala i sistemsko programiranje,
Vjerojatnost i statistika, Numeričke metode u računarstvu

"Cijelo je vrijeme naglasak na matematici, ne na programiranju, a još manje na elektronici."

"Primjer teme diplomskog rada: Model napadača u nekom protokolu."

ETH–Zürich, Grgur Petric Maretić

G. Petric Maretić, Fuzzy logika, Diplomski rad, PMF–MO, 2010.

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Matematička analiza 1

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"Primjer teme diplomskog rada: Model napadača u nekom protokolu."

Philadelphia, Andrej Scedrov (1)

Computer Science

Prva godina: Introduction to Programming, **Calculus I&**, Principles of Physics I, Natural science lab, Social Sciences and Humanities elective (SSH), Free elective, Programming Languages and Techniques I, **Math Foundations of Computer Science**, Natural science elective

Druga godina: Introduction to Computer Architecture, Principles of Physics II, Discrete Probability, Stochastic Processes, and Statistical Inference or Engineering Probability or Engineering Statistics, Natural science lab, Two SSH electives; Programming Languages and Techniques II, Digital System Organization/Design and lab, **Math elective**, Two SSH electives

Philadelphia, Andrej Scedrov (1)

Computer Science

Prva godina: Introduction to Programming, **Calculus I&**, Principles of Physics I, Natural science lab, Social Sciences and Humanities elective (SSH), Free elective, Programming Languages and Techniques I, **Math Foundations of Computer Science**, Natural science elective

Druga godina: Introduction to Computer Architecture, Principles of Physics II, Discrete Probability, Stochastic Processes, and Statistical Inference or Engineering Probability or Engineering Statistics, Natural science lab, Two SSH electives; Programming Languages and Techniques II, Digital System Organization/Design and lab, **Math elective**, Two SSH electives

Philadelphia, Andrej Scedrov (2)

Treća godina: **Automata, Computability and Complexity**, Computing Operating Systems and Lab, CSE elective, Technical elective, Intro to Algorithms, Two CSE electives, Technical elective, **Math elective**

Četvrta godina: Senior Design I, Two technical electives, SSH elective, Free elective, Senior Design II Two technical electives, Free elective

<http://www.cis.upenn.edu/ugrad/>

Philadelphia, Andrej Scedrov (2)

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<http://www.cis.upenn.edu/ugrad/>

Ljubljana, Andrej Bauer (1)

Interdisciplinarni univerzitetni študijski program Računalništvo in matematika

Studij traje tri godine.

Prva godina: Analiza I&II, Diskretne strukture I&II, Osnove digitalnih vezij, Osnove programiranja, Linearna algebra, Programiranje in algoritmi, Osnove podatkovnih baz.

Druga godina: Analiza III, Kombinatorika, Algoritmi in podatkovne strukture, Arhitektura računalniških sistemov, Osnove umetne inteligence, Izbrana poglavja iz matematike, Optimizacijske metode, Principi programskih jezikov, Operacijski sistemi, Računalniške komunikacije

Ljubljana, Andrej Bauer (1)

Treća godina: Numerične metode, Komuniciranje in vodenje projektov, Verjetnostni račun in statistika, Modul, Strokovni izbirni predmet, Diplomski seminar, Splošni izbirni predmet

<http://www.fmf.uni-lj.si/si/studij-matematike/interdisciplinarni-univerzitetni-studijski-program-racunalnistvo-in-matematika/>

Ljubljana, Andrej Bauer (1)

Treća godina: Numerične metode, Komuniciranje in vodenje projektov, Verjetnostni račun in statistika, Modul, Strokovni izbirni predmet, Diplomski seminar, Splošni izbirni predmet

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